Public Employment, Competition, and Economic Growth

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Abstract

This paper shows that there is an inverted U-shaped relationship between public employment and economic growth. The government allocates workers to the production of public goods, which reduces operational costs, increasing the number of firms in the market, and creating incentives to innovate. Conversely, large public sectors crowd out the labor market, reducing the number of firms and the incentives to innovate. An extension of the model shows that the average human capital of public workers has the same relationship with economic growth. Therefore, even countries with small public sectors can hinder economic growth by hiring many high-productivity workers. The paper also provides empirical evidence of an inverted U-shaped correlation between the share of public workers and economic growth using data from the Worldwide Bureaucracy Indicator and the Penn World Tables. Numerical exercises show that the model replicates the inverted U-shaped relationship between public employment and economic growth found in the data.

Keywords: Public Employment; Innovation; Competition; Endogenous Growth
JEL: J45; J24; O30; O40

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1 Introduction

Government participation in economic activity is a vastly studied topic in economics. The debate ranges from the optimal size of the public sector to its optimal role in the economy. An undisputed fact in the literature is that government participation in economies has been increasing throughout the last century, especially before 1980 (Tanzi and Schuknecht, 2000). Average general government spending as a share of GDP for industrial countries increased by a factor of 3.5 from 1913 to 1990 (Tanzi and Schuknecht, 1997). Consequently, there exist many studies investigating the relationship between government size and economic growth.

Nonetheless, most of these works focus on the interplay between economic growth and public expenditure, tax revenue, or public investment in either infrastructure or R&D. In other words, they investigate the impact of government participation on the market for final goods. This literature suggests an inverted U-shaped relationship between economic growth and government size.[1] In contrast, this paper provides theoretical bases for this relationship by exploring the government's effects on the labor market. The key element in the theory is the relationship between the size of the labor force available to private firms, competition, and the incentives to innovation.[2] As a result, this paper puts forth an endogenous growth model where public employment has a nonlinear relationship with economic growth.

The model features an economy where firms produce differentiated goods using only labor as input. The government can increase competition in the private sector by reducing the operational costs of production by providing cost-saving services.[3] On the other hand, the government dampens competition by crowding out the supply of labor in the private sector. As a result of competition, firms engage in the production of new technology, creating a link between government size and economic growth. The inverted U-shaped relationship between public employment and economic growth follows directly from this mechanism. For low levels of public employment, the government provides few public goods and, consequently, firms demand many workers to perform operational tasks. The need for more workers reduce the number of firms in

[1] This curve is known as Armey Curve after Armey (1995). Also referred to as the BARS curve after Barro (1990), Armey (1995), Rahn and Fox (1996) and Scully (1989); or even the Scully Curve.

[2] The relationship between market size and economic growth was proposed by Desmet and Parente (2010). Desmet and Parente (2012) use this theory to explain the transition to sustained growth.

[3] For example, the government can allocate workers to enforce contracts and guarantee the well functioning of institutions, which would imply the reduction of transaction costs (Krasa and Villamil, 1992, 1994).

the economy, weakening competition, and reducing incentives to innovate. As the size of public employment increases, operational costs in terms of workers decrease. Then, the number of firms increases and tougher competition creates incentives to innovate. Eventually, the decline of operational costs does not compensate for the crowding out of workers in the labor market. In this situation, increasing government size hinders competition and economic growth.

I provide evidence of an inverted U-shaped relationship between the share of public workers and economic growth predicted by the model. The percentage of public workers is measured using the Worldwide Bureaucracy Indicator collected by the World Bank, whereas data on real GDP per capita is from the Penn World Table 9.1 (Feenstra et al., 2015). The quadratic relationship persists even when I control for initial conditions and when I use an alternative definition of public employment. I also provide evidence of the effectiveness of public workers in delivering public goods and reducing the cost of business. Using a measure of public goods utilized by Desmet et al. (2012, 2017), I show that countries with a larger share of public workers are associated with better provision of public goods. I also create a measure of business cost using individuals' perceptions collected by World Economic Forum in the Global Competitiveness Report. Again, larger public sectors are associated with lower business costs. Both results stand when I control for the level of development of the countries. Numerical exercises show that the model replicates the inverted U-shaped relationship found in the data.

Having established the relation between public employment and economic growth, I extend the model to account for human capital. I show that the average human capital of public workers acts as a substitute for the share of public workers. Countries with a relatively small number of public employees may still create a sizeable crowding-out effect when the average human capital of public workers is well above the average human capital of the economy. Then, I use the data from the Worldwide Bureaucracy Indicator to show that the inverted U-shaped curve stands when the measure of public employment is adjusted by the average human capital of public workers.

This paper is related to the recent literature that investigates the effects of public employment on economic development. Gomes and Kuehn (2017) is a recent closely related work that considers the reallocation of human capital from the private to the public sector. More extensive public employment increases the average firm size as higher wages induce entrepreneurs to become paid workers. The smaller number of entrepreneurs reduce output by a small amount. Similarly, Cavalcanti and dos Santos (Forthcoming) analyze the effects of occupational choices distorted by the public wage premium on countries' productivity through on-the-job accumulation of human capital. The paper finds that a reduction in the wage premium would result in a sizable increase in output. Jaimovich and Rud (2014) also show that the wage premium resulting from a bloated public sector leads to lower profits and undermines private incentives to produce.

The present paper contributes to the above literature in at least two points. First, I provide an endogenous growth model where public employment is a crucial element to explain relative growth disparities, whereas the papers discussed above focus on relative income disparities. Second, the model proposed in this paper does not rely on the public wage premium as a source of misallocation of factors. In equilibrium, wages are equal, and workers are randomly sorted between sectors. Large governments simply reduce the size of private markets, which reduces the incentives to innovate.

This paper also talks to the literature on the interaction between public and private sectors on the labor marker.[4] Algan et al. (2002) and Behar and Mok (2019) are empirical papers that stress the crowding-out effect of the government on the labor market. More recently, Becker et al. (2018) study the surge in public employment in the city of Bonn after the transfer of the federal government from Berlin. The authors find an overall small positive effect on employment in the private sector. However, they credit this result to two offsetting effects. The larger public sector reduced employment in industry, but generated amenities that increased employment in other sectors. The mechanisms identified by Becker et al. (2018) are similar to the ones I model in this paper. The government has a positive effect on the economy by providing public goods, but it also has negative effects when it crowds out the labor market.

The paper is structured as follows: After the introduction, Section 2 presents the model, whereas Section 3 presents empirical evidence on the relationship between public employment, economic growth, and the provision of public goods. Section 4 pertains to the a numerical exercise. In Section 5, I extend the model to account for the human capital of public workers. Section 6 concludes the paper.

[4]There is also a vast literature that models the interaction between public and private sectors to understand the public wage premium. See, for example, Domeij and Ljungqvist (2019) and Garibaldi et al. (2020).

2 Public employment, competition, and innovation

In the model economy presented in this section, consumers have one most preferred variety, and private firms produce one single variety of goods and choose the number of workers they will assign to the production of new technologies. Therefore, as competition increases and more varieties enter the market, firms face more sensitive demands for their goods, reduce the prices, and need to be more innovative to break even. The government provides cost-saving services for private-sector production using workers as input.

Whereas the model proposed by Desmet and Parente (2010) links the number of firms in the market and innovation, the present paper links the government size with the number of firms in two ways: by the decreasing cost of production and by crowding out the labor market. These two effects combined are key for the conclusion that there is an inverted U-shaped relationship between government size and economic growth. In Section 5, I also add human capital to the model.

2.1 Government

The government provides cost-saving services G using workers, N^p, as input using the technology $F(\cdot)$. I assume that $F(\cdot)$ is increasing in N^p and has constant returns to scale. Therefore, one can write the average provision of public goods as a function of the share of public workers, s^p, as

$$g = F(s^p).$$

The firms absorb the services g provided by the public sector to reduce their operational costs. The government pays the public workers the wage w^g, financed by a flat-rate tax, τ, on labor income such that,

$$w^g s^p N = \tau Y, \tag{1}$$

where Y is the aggregate income and N is the labor force.

2.2 Firms

The firms produce differentiated goods indexed by ν. Each firm hires N_ν units of labor, which is the only input of production. Workers can be used either in production activities or in nonproduction activities such as technology creation or operational tasks. Given the technology A_ν, the variety-ν firm produces:

$$Q_\nu = A_\nu[N_\nu - \kappa_\nu(g)], \tag{2}$$

where $\kappa_\nu(g)$ is the amount of workers allocated to non-production activities, which has two parts given by

$$\kappa_\nu(g) = \kappa(g)e^{\phi\gamma_\nu}.$$

The first part, $\kappa(g)$, is variety-independent and a function of the pubic good g. Thus, $\kappa(g)$ is a cost in terms of labor that every firm, no matter the variety, must pay to produce. It represents the amount of labor that every firm must allocate to, for example, prepare its taxes and enforce contracts. Therefore, the more help the public sector provides in the form of public services, the lower is the number of nonproduction workers firms need to operate. For example, the public sector can allocate employees to enforce the contracts reducing the intermediation costs, which has been shown by Antunes et al. (2008) to explain international income disparities.

The second term is firm-specific and relates to the size of innovation γ_ν chosen. This part of the cost is also not directly productive. It is the number of workers the firm allocates to the production of better technologies such that the firm's productivity is given by

$$A_\nu = (1 + \gamma_\nu)A_x,$$

where A_x is the benchmark technology, which is the average technology used when the production starts. The variety-ν firm's profit is given by

$$\Pi_\nu = p_\nu C_\nu - w^x N_\nu. \tag{3}$$

where C_ν is the aggregate demand for variety ν, p_ν is its price and w^x is the wage paid to unit of labor. The firm chooses price and the size of technological innovation to maximize Eq. (3). The first-order condition with respect to prices yields

$$p_\nu = \frac{w^x}{A_x(1+\gamma_\nu)} \frac{\varepsilon_\nu}{\varepsilon_\nu - 1}, \tag{4}$$

which is a function of the price-elasticity of demand $\varepsilon_\nu = -\frac{\partial C_\nu}{\partial p_\nu} / \frac{p_\nu}{C_\nu}$. The first-order condition with respect to the size of technology innovations is

$$-\phi\kappa(g)e^{\phi\gamma_\nu} + \frac{C_\nu}{A_x(1+\gamma_\nu)^2} \leqslant 0 \tag{5}$$

where the inequality holds only for the case where γ_ν is not an interior solution.

2.3 Households and Aggregate Demand

Households are heterogeneous in their most preferred variety. There is a continuum of size N of households uniformly distributed in a circle with circumference one. The household whose most preferred variety is $\tilde{\nu}$ chooses the amount of each variety and its job to maximize the following utility function:

$$U_{\tilde{\nu}}\left(\{c_\nu\}_{\nu \in V}, a\right) = \max_{\nu \in V} \left[\frac{c_\nu}{1 + d_{\nu\tilde{\nu}}^{\beta}} \right], \tag{6}$$

subject to

$$\sum_{\nu \in V} p_\nu c_\nu \leqslant (1-\tau)\left[a w^g + (1-a) w^x\right],$$

where τ is the labor income tax and w^g is the wage in the public sector. The occupational choice is given by the variable $a \in \{0, 1\}$, where $a = 0$ if the household chooses to work in the private sector, and $a = 1$ otherwise. The occupational choice depends uniquely on the wages offered.[5]

[5]The utility function described above is of the type Hotteling-Lancaster. As discussed in Desmet and Parente (2012, p. 212), it is not the only type of utility function that would delivery the key relationship between market size and innovation. However, it has friendly properties that assist with the understanding of the mechanisms.

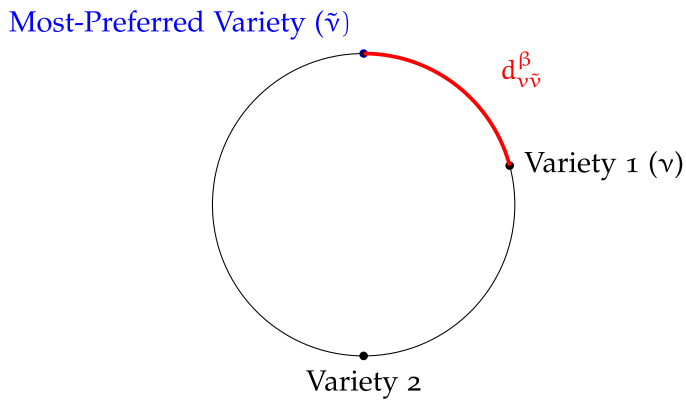

Figure 1: Variety discount

The utility of the household is given by the amount of consumption of the variety whose consumption, discounted by $1 + d^{\beta}_{v\tilde{v}}$, is the greatest. As illustrated by Fig. 1, $d_{v\tilde{v}}$ is the arc-distance between the varieties v and $\tilde{v}$, and β is how much this distance affects utility. When $\beta = 0$, there is no discount from consuming a variety that is not the most preferred one. Note from the utility function that there is no reason why a household would choose to consume more than one variety. Specifically, the variety consumed by household $\tilde{v}$ is the one with smaller cost, in terms of prices and distance to the most preferred variety,

$$v' = \text{argmin}[p_v(1 + d^{\beta}_{v,\tilde{v}})|v \in V],$$

where V is the set of varieties produced. Individuals spend all their disposable income on their chosen variety such that

$$p_{v'}c_{v'} = \begin{cases} (1-\tau)w^g & \text{if } w^g > w^x \\ (1-\tau)w^x & \text{if } w^g \leqslant w^x. \end{cases} \tag{7}$$

The aggregate demand for each variety depends on the number of households that demands each variety, which depends on the prices and distance to the most preferred variety.[6] As defined below, the equilibrium in this economy is a symmetric Nash equilibrium where all m varieties produced are equally spaced around the varieties circle, so that $d = 1/m$ is the arc-distance

[6]In equilibrium, I assume $w^g = w^x$ such that individuals are indifferent between sectors. This assumption allows us to focus on the effects of the public sector free from the public-wage-premium effect.

between any two produced varieties. Then, the household who is indifferent between variety k and an adjacent variety j is the one located at a distance d_k^i from k, such that

$$p_j[1+(d-d_k^i)^\beta] = p_k[1+(d_k^i)^\beta]. \tag{8}$$

Therefore, aggregating Eq. (7) for twice the fraction d_k^i of the continuum of households, since the condition above holds for both varieties neighboring k, the aggregate demand of variety k is

$$C_k = \frac{(1-\tau)2d_k^i}{p_k}\left[\int_0^N a(h)w^g dv + \int_0^N (1-a(h))w^x dv\right]. \tag{9}$$

The aggregate demand for variety k depends on the measure d_k^i, which changes with the price of the variety k. It is true because the individual who is indifferent to the variety k changes when prices change. According to the equilibrium condition Eq. (8) the price-elasticity of demand is

$$\varepsilon_k = 1 + \frac{(1+d_k^i)^\beta[1+(d-d_k^i)^\beta]}{\beta(d-d_k^i)^{\beta-1}[1+(d_k^i)^\beta]d_k^i + \beta(d_k^i)^\beta[1+(d-d_k^i)^\beta]}. \tag{10}$$

2.4 Equilibrium

The symmetric equilibrium is defined below.

Definition 1 (Symmetric Equilibrium)**.** *The symmetric equilibrium is a vector of variety-dependent variables* $\{p_v, d_v^i, \varepsilon_v, Q_v, \gamma_v, C_v\}_{v\in V}$ *for all* $v \in V$ *and a vector* $\{d, w^x, w^g, \tau\}$ *that, given* $\{N, s^p\}$*, satisfy:*

1. *Firm maximization conditions Eqs.* (4) *and* (5)*, and the zero profit condition [Eq.* (3)*=0] for all* $v \in V$*;*
2. *The aggregate demand Eq.* (9) *for all* $v \in V$*;*
3. *The government balanced budget condition Eq.* (1)*;*
4. *The symmetric equilibrium price condition Eq.* (8) *for all* $v \in V$*;*
5. *The elasticity Eq.* (10) *for all* $v \in V$*;*
6. *The good's market clearing:* $Q_v = C_v$ *for all* $v \in V$*;*
7. *Labor market equilibrium wages:* $w^x = w^g$*; and*

8. The labor market clearing: $\frac{1}{d}N_v = (1 - s^p)N$ *for all* $v \in V$.

One can use Eqs. (1) and (9), and the labor market equilibrium wages, to rewrite the aggregate demand as

$$C_k = \frac{2d_k^i w^x}{p_k}(1 - s^p)N. \tag{11}$$

This is the crowding-out effect that explains the negative effect of government on economic growth. Larger governments reduce the supply of labor in the labor market reducing firm's output.

The symmetry conditions $p_j = p_k$ for all $j, k \in V$ and $d = 2d_k^i$ together with Eq. (10) yield the equilibrium equation for the price-elasticity of demand

$$\varepsilon_k = 1 + \frac{1}{2\beta}\left(\frac{2}{d}\right)^\beta + \frac{1}{2\beta}, \tag{12}$$

which depends on the number of firms in the market $m = 1/d$. In its turn, the number of firms can be found using the final goods market clearing condition together with the firm's first-order condition Eq. (5), and the private production function such that $N_k = \kappa(g)e^{\gamma_k \phi}\varepsilon_k$. It can be used together with the labor market clearing condition to find

$$m = \frac{1}{d} = \frac{(1 - s^p)N}{N_k} = \frac{(1 - s^p)N}{\kappa(g)e^{\gamma_k \phi}\varepsilon_k}. \tag{13}$$

Government size affects the number of firms in two ways: the first is the crowding-out on the labor market mentioned above; the second is by reducing the costs of production in terms of labor force. With reduced costs, there will be more firms in the economy, what increases the price-elasticity of demand in Eq. (12).

One can find the equilibrium equations for the model putting together Eqs. (12) and (13) to find a negative relationship between elasticity and innovation, and use the zero profit condition and Eq. (5) to find a positive relationship

$$\varepsilon_k = 1 + \frac{1}{2\beta}\left[\frac{2(1 - s^p)N}{\kappa(g)e^{\phi\gamma_k}\varepsilon_k}\right]^\beta + \frac{1}{2\beta} \quad \text{and} \quad \gamma_k = \frac{\varepsilon_k - 1}{\phi} - 1.$$

Therefore, there is an equilibrium value for the innovation process that satisfies the following equation

$$[2\beta\phi(1+\gamma_k)-1]e^{\beta\phi\gamma_k}[1+\phi(1+\gamma_k)]^{\beta} = \left[\frac{2(1-s^p)N}{(\kappa \circ F)(s^p)}\right]^{\beta}. \tag{14}$$

and depends on the size of government. The following proposition summarizes this equilibrium relationship.

Proposition 1. *In the symmetric equilibrium, if the elasticity of the operational costs to the share of labor in the public sector is bounded, the relationship between economic growth and the share of labor in the public sector is nonlinear. Specifically, it is an inverted U-shaped function.*

Proof. It follows directly from equation Eq. (14). See Appendix A.2 for details. □

Proposition 1 formalizes the main result of the theory. For small levels of s^p, there is little production of public goods, and by increasing it, the country may reduce the costs of operation, which should have a more considerable gain in terms of innovation than the reduction of labor available for the private sector. Thus, the effect of more workers allocated into the public sector would be positive on γ_k. However, for high levels of s^p, the marginal benefit of a public workers is minimal, and the gains from reducing a firm's costs are lower than the loss from the crowding-out effect. This scenario leads to a negative relationship between s^p and innovation. Importantly, in equilibrium, the innovation chosen by firms is the same. Then, A_x grows at a rate γ_v. That is, γ_v is the growth rate of the economy.

3 Empirical Evidence

I provide cross-country empirical evidence of the nonlinear relationship between public employment and economic growth using data on public employment from the Worldwide Bureaucracy Indicator (WWBI). This database, organized by the World Bank, uses microdata from the World Bank's International Income Distribution Database, the Luxembourg Income Study, and the International Comparisons Program wage survey to create cross-country comparable data on public wages, employment, and wage bill. Public employees are defined as workers in state-owned institutions. This definition includes state-owned enterprises.[7] The share of public workers, s^p, is defined as the ratio to total employment. I use data on real output per capita from the Penn World Table 9.1 (PWT9.1) to show the relationship between public work and economic growth (Feenstra et al., 2015).[8]

The WWBI database is an unbalanced panel of 115 developing countries ranging from 2000 to 2016. The wealthiest country in the sample is Slovenia, whereas the poorest is the Democratic Republic of Congo. I average both the share of public workers and the growth rate of real output per capita to achieve a cross-section of 97 countries.[9] The average share of public workers in the sample is 15.4%, where Bosnia and Herzegovina has the largest share (45.2%), and Haiti has the lowest (2.5%). In terms of growth rates, the average for the sample in the period was 3.4% per year. Nigeria is well above it with an average growth rate of 9.4% per year, whereas Zimbabwe performed a negative average growth rate of -2.2% per year.

Fig. 2 shows that countries with low shares of workers allocated to the public sector experienced a slow average growth of output per capita between 2000 and 2016. As the share of public employment increases, the growth rates also increase but at diminishing rates. At some level, increasing the share of public employment seems to be detrimental to economic growth. It characterizes the inverted U-shaped relationship stressed by the quadratic fit in the figure. This nonlinear relationship stands when I control for initial conditions often considered as determinants of growth. Specifically, I control for the initial log of real output per capita and initial average human capital. These results are in Table A.1. The simple correlation depicted in Fig. 2 predicts an optimal size of public employment equal to 27.3%. When I control for the initial level of development, this number is unchanged.

Fig. 3 provides evidence of the relationship between public employment and provision of cost-saving public goods. The data on public goods is from Desmet et al. (2012, 2017). The authors

[7]In Appendix A.3, I repeat the analysis using public employment defined as workers allocated in the production of government services. The data is from the sectoral database constructed by Timmer et al. (2015). This alternative database also extends the current analysis by covering developed and developing countries for a longer period of time.

[8]A better empirical counterpart of γ_v would be the growth of output per worker instead of the growth of output per capita. However, the results are unchanged if I use output per worker instead of output per capita.

[9]It is impractical to exploit the time-series feature of the data since it is too short of a panel with very few countries showing up at least three times. Additionally, there is a modest variation in the size of government within countries. Importantly, Hauk and Wacziarg (2009) show that a cross-section analysis of averaged variables performs better in growth regressions.

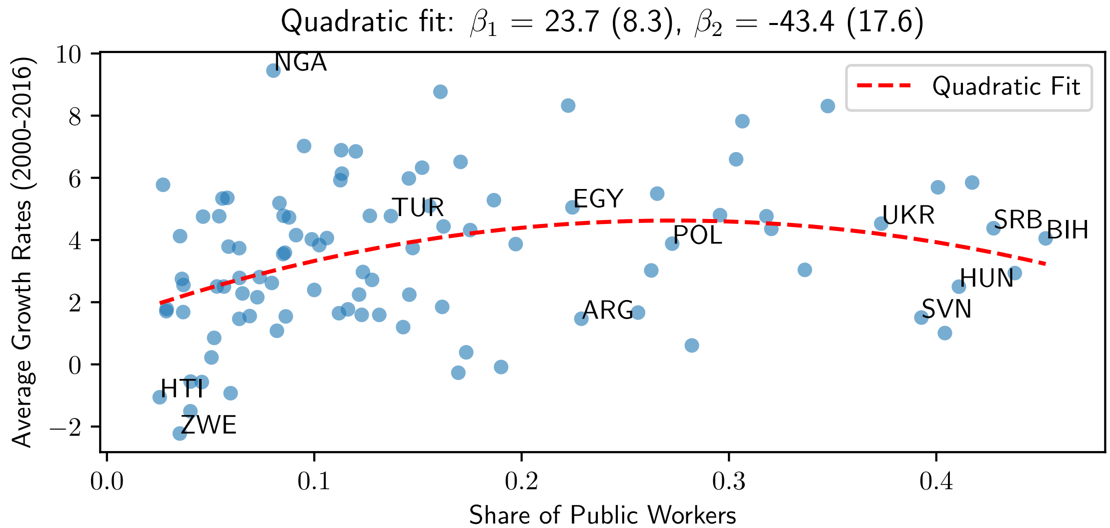

Figure 2: Economic growth and share of public workers

Note: Data on public employment is from the WWBI. Growth of GDP per capita is from PWT9.1 (Feenstra et al., 2015). The values are averages of 2000 to 2016. The quadratic fit fits $\gamma_i = \alpha + \beta_1(s_i^P) + \beta_2(s_i^P)^2$. Robust standard errors in parentheses.

use the principal component of 8 World Development Indicators as a measure of public goods. Specifically, it is the principal component of infant mortality, measles immunization rate, hospital beds per capita, log school attainment, percentage access to improved sanitation, percentage access to improved water, a measure of infrastructure quality, and railway length per capita. The data on public goods is averaged between 1990 and 2010. Hungary is the country with better provision of public goods, whereas Mali has the lowest value. As depicted in Fig. 3a, countries with higher shares of workers allocated into the public sector provide the society with better public goods as stressed by the linear-log fit in the figure. This relationship holds when I control for both the average log of public capital per capita and the average log of output per capita.

I also use the perceptions of costs imposed on business to construct a business cost index. This index measures how costly is the lack of public goods for private companies using data collected by the World Economic Forum in the Global Competitiveness Report between 2007 and 2017. Specifically, the business cost index is the average of 7 indexes: the burden of government regulation, business costs of terrorism, business costs of crime and violence, quality of overall infrastructure, the business impact of malaria, the business impact of tuberculosis, and quality of the education system. The final index inverts and re-scales the original indexes such that

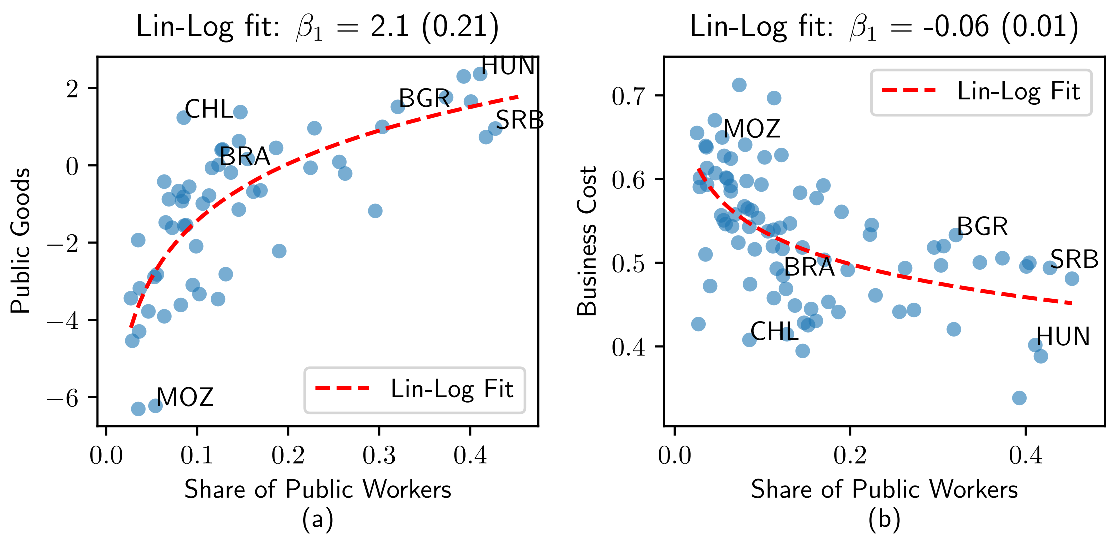

Figure 3: Public goods, business cost, and the share of public workers

Note: Data on public employment is from WWBI. The measure of public goods is from Desmet et al. (2017). It is the principal component of 8 indicators: infant mortality, measles immunization rate, hospital beds per capita, log school attainment, percentage access to improved sanitation, percentage access to improved water, a measure of infrastructure quality, and railway length per capita. The variable are averages between 1990 and 2010. The Log-Linear model fits public goods$_i = \alpha + \beta_1 \ln(s_i^p)$. The business cost index is the average of 7 indexes: the burden of government regulation, business costs of terrorism, business costs of crime and violence, quality of overall infrastructure, the business impact of malaria, the business impact of tuberculosis, and the quality of the education system. The final index inverts and re-scales the original indexes such that larger values imply higher costs. The Log-Linear model fits business cost$_i = \alpha + \beta_1 \ln(s_i^p)$. Robust standard errors in parentheses.

larger values mean higher costs. Slovenia is the country in the sample with the lowest business costs, whereas Chad is the country with the largest burden imposed on business. Fig. 3b depicts the relationship between business cost and the share of public workers. A larger share of employees allocated to the public sector decreases the index of business costs. This result is unchanged when I control for the output per capita of countries.

The facts above serve as empirical evidence for a key mechanism in the theory presented in Section 2. Namely, the existence of the government is crucial for private production as it provides public goods that reduce the cost of production.

4 Numerical Analysis

In this section, I perform a numerical exercise to test how well the model fits the cross-country data discussed previously. Although the model is not appropriate to perform a complete quantitative exercise, the following experiment serves as a test of the model. Numerical experiments require the choice of functional forms for $F(\cdot)$ and $\kappa(\cdot)$, and the choice of parameters β, ϕ, and N. Both $F(\cdot)$ and $\kappa(\cdot)$ are assumed to have constant elasticities to their arguments. Specifically, $F(h^p) = (s^p)^\alpha$ and $\kappa(g) = g^\eta$. α is set equal to 1 such that the government also has a linear production function. I follow Desmet and Parente (2012) on the choice of $\beta = 0.5$ to match the mark-ups estimates from Jaimovich and Floetotto (2008).[10]

The choice of η and ϕ requires more explanation. One can show that η and ϕ are the parameters that determine the optimal size of public employment. Therefore I choose η to match the optimal size of public employment, whereas ϕ matches the growth rate at optimum. I follow two approaches to choose the numbers η and ϕ match. First, I choose optimal employment equal to 27.3% and an optimal growth rate of 4.6%, as estimated in Fig. 2. Then, the model must match the peak of the quadratic fit. I call this exercise *match WWBI*. In the second approach, I use values computed by Facchini and Melki (2013) and Rose and Page (1985) for France. Facchini and Melki (2013) use data from 1896 to 2008 to estimate the optimal size of government

[10]The experiments are not sensitive to the choice of α since what matters for optimal growth is the elasticity of the composite function $\kappa \circ F$. The choice of α will affect only the value of η. The sensitiveness to the choice of β is small. Smaller β leads to smaller mark-ups and more incentive to innovation. Larger β has the opposite effect. However, it does not change the main picture.

spending is 30% of GDP, what happened in 1980.[11] Using a similar definition of public employment as the WWBI, Rose and Page (1985, Table 3.14, p. 122) estimate a share of public workers of 29.1% in 1980 in France. This number is close to the 27.3% estimated using the WWBI database in Section 3. Therefore, the second exercise sets η to match an optimal public employment of 29.1%, and ϕ to match the growth at optimum equal to 3.2% as estimed by Facchini and Melki (2013). I call this exercise *match France*.

Fig. 4 displays the predicted growth rates for countries presented in Fig. 2 according to the quadratic fit and both exercises. The figure shows that the model captures the main nonlinear pattern in the data. I compare the three models using the sum of squared errors (SSE). The quadratic fit has the lowest SSE equal to 669 and serves as a reference since it seeks to best match the sample. The *match WWBI* exercise has SSE equal to 737, which is very close to the quadratic fit, given that the only value matched is the optimal size of public employment. Note that this exercise practically overlaps the quadratic fit for high levels of public work. For low levels of public employment, the model predicts a faster decline in growth rates compared to the data. This rapid decay matches the experience of several countries but fails to explain significant growth rates observed in many countries with small governments. In the model, this is partially explained by the fixed cost of technology adoption ϕ. Based on the literature of technological catch-up, developing countries have lower costs of technology adoption (Barro and Sala-I-Martin, 1997; Griffith et al., 2004; Acemoglu et al., 2006). Adding this to the model would certainly account for the fast-growth-small-government cases.

The *match France* exercise has SSE equal to 1489, which is well above the SSE of the quadratic case. However, given that this exercise matches out-of-sample data of a country richer than all countries in the sample, this SSE is very close to the quadratic fit. In fact, one can note that this exercise is only a downward shift of the *match WWBI* exercise since both simulations use similar optimal sizes of public employment. However, they differ significantly on the optimal growth reached optimally. Again, the reason for that is the higher ϕ in France than the countries in the sample.

[11] The estimation of the optimal size in terms of employment is rare. One example is dos Reis et al. (2016) that computes the optimal employment for the Brazilian economy equal to 13.5% of the workforce. They calibrate a model where the public sector can reduce uncertainty making larger public sectors more desirable.

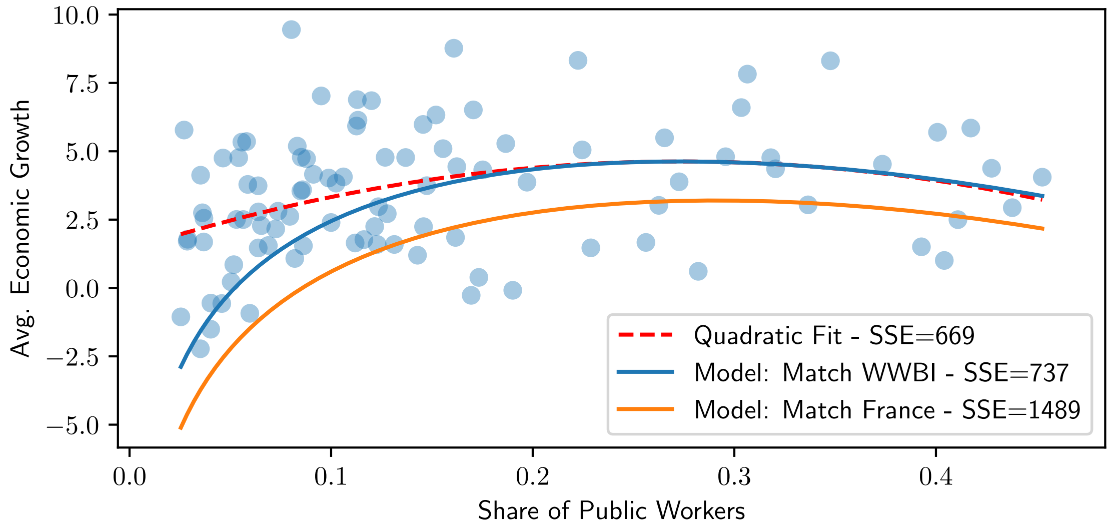

Figure 4: Numerical Exercise

Note: Same data as in Fig. 2. The *match WWBI* model has α=1, β=0.5, z=1, N=1, ϕ=1.50, η=-0.38. *Match France* has α=1, β=0.5, z=1, N=1, ϕ=1.51, η=-0.41.

5 Human Capital

In this section, I extend the model from Section 2 to include human capital.[12] This extension is interesting as it highlights that the potential of the average human capital of public workers to act as a substitute for the share of public workers. That is, countries may compensate for small public sectors with highly educated individuals. Although these workers can provide more cost-saving services, by hiring them, the government is also crowding-out the market for human capital.

The model has public and private production functions as functions of efficient units of labor such that the provision of public goods depends on the share of workers in the public sector adjusted by the average human capital of these workers, denoted by h^p. There is a continuum of measure N of varieties uniformly distributed in a circle with circumference one. In each variety, there is a continuum of measure one of households. They draw their labor productivity, h, from $\mathcal{H}$ according to the probability density function $q(\cdot)$ such that total stock of human capital in the

[12] Here I discuss only the main changes in the benchmark model in Section 2. The full model with efficient labor is in Appendix A.4.

economy is

$$H = \int_0^N \int_{h \in \mathcal{H}} h q(h) dh d\nu.$$

The draws of the labor productivity variety independent, which means that high-productivity workers are as likely to prefer one kind of good as low-productivity workers. This assumption abstracts from any unbalanced increase in the number of high-productivity-workers-biased goods.

After the adjustments above, one can follow the same steps as in Section 2 to find the following relationship between the average human capital in the public sector and economic growth,

$$[2\beta\phi(1+\gamma_k) - 1]e^{\beta\phi\gamma_k}[1+\phi(1+\gamma_k)]^\beta = \left[\frac{2(H - h^p s^p N)}{(\kappa \circ F)(h^p s^p)}\right]^\beta. \tag{15}$$

Eq. (15) stresses that the average human capital of workers in the public sector is a substitute for the share of workers. This fact is true both for the provision of public goods and the crowding out in the labor market. Intuitively, if the share of public workers is constant, the provision of public goods increases with the average human capital of the public workers. However, given the total stock of human capital in the labor market, hiring high-productivity public workers translates into a smaller supply of human capital available for the private sector. It weakens competition and hinders economic growth. Thus, it is natural to extend Proposition 2 to the average human capital of public workers.

Proposition 2. *In the symmetric equilibrium, if the elasticity of the operational costs to the share of public workers adjusted to their average human capital is bounded, the relationship between economic growth and the average human capital of public workers is nonlinear. Specifically, it is an inverted U-shaped function.*

Proof. It follows directly from equation Eq. (15). See Appendix A.2 for details. □

Given this result, I show that the results found in Section 3 also hold for the adjusted share of public workers $h^p s^p$. The WWBI provides information about the number of public workers by education groups. To compute the average human capital among public workers, I follow the economic growth literature by considering the stock of human capital as an exponential function of the average years of education yrs,

$$h = e^{re \times yrs},$$

where re is the returns to education, which I set equal to 0.1.[13] I use data on duration of each education group from UNESCO to build the average years of schooling of public workers.[14]

Fig. 5 shows the relationship between the human capital of public workers relative to the human capital of private workers and economic development. First, one can observe that, for the vast majority of countries in the sample, the average human capital of public workers is above the average human of private workers. This bias towards educated workers is in accordance with evidence provided by, for example, Fontaine et al. (2020). There is a negative association between these two variables. For instance, the average human capital of public workers is 91% higher than the average human capital of private workers in Malawi. For richer countries, this number declines considerably. For example, Serbia has public workers with average human capital 10% lower than the average human capital of private workers.

Using this data, we provide evidence similar to the one presented in Section 3 using the share of public workers adjusted by the average human capital in the sector. Fig. 6a shows the inverted U-shaped correlation between average growth and the adjusted share of public workers. There are 76 countries in the sample, and the simple quadratic fit explains 17% of the variation in the average growth of these countries. The results are unchanged when we control for initial conditions using the log of per capita output and average human capital. Refer to Table A.2 in Appendix A.1 for full regression tables.

Fig. 6b depicts the positive relationship between the adjusted share of public workers and the production of public goods. As in Section 3, there is a strong positive relationship between the size of the public sector and the production of public goods. In this sample of 48 countries, the log of the adjusted share of public workers explains 62% of the variation in the provision of public goods. This result stands when we control for the log of public capital

[13] See, for example, Hall and Jones (1999) and Bils and Klenow (2000).

[14] Lack of information on completion would require adjustments as ones made in Lutz et al. (2007). However, since public employment usually requires a minimum level of education for each position I do not pursue these corrections.

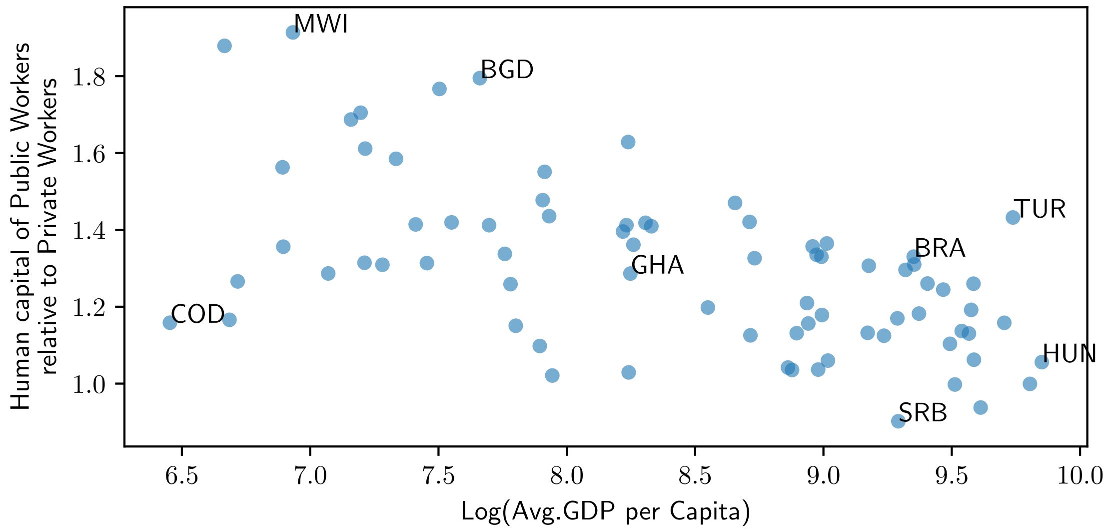

Figure 5: Average human capital of public workers relative to paid-private workers

Note: Data on public employment and private employment by education group is from WWBI. GDP per capita is from PWT9.1. The values are averages of 2000 to 2016

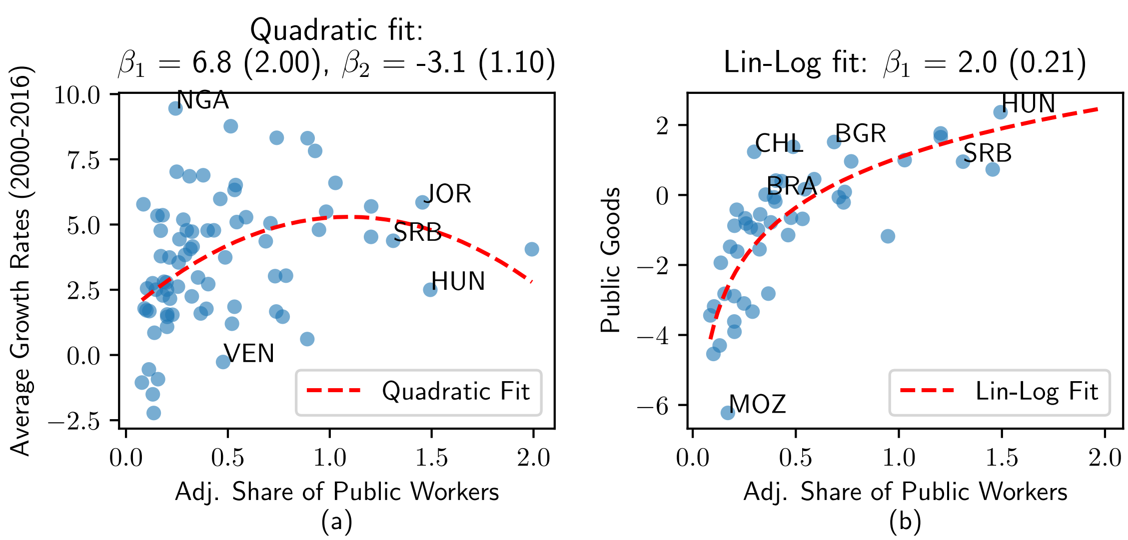

Figure 6: Economic growth, public goods, and adjusted share of public workers

Note: Data on public employment by education group is from WWBI. Growth of GDP per capita is from PWT9.1. The values are averages of 2000 to 2016. The measure of public goods is from Desmet et al. (2017). It is the principal component of 8 indicators: infant mortality, measles immunization rate, hospital beds per capita, log school attainment, percentage access to improved sanitation, percentage access to improved water, a measure of infrastructure quality, and railway length per capita. The public goods indicators are averaged between 1990 and 2010. In panel (a), the quadratic fit fits $\gamma_i = \alpha + \beta_1(h_i^p s_i^p) + \beta_2(h_i^p s_i^p)^2$. In panel (b), the Lin-Log fit fits $g_i = \alpha + \beta_1 \ln(h_i^p s_i^p)$. Robust standard errors in parentheses.

stock per capita and log of per capita output. The coefficient declines to 1.2 (s.e.=0.24) after controlling for public capital and level of development.

The discussion provided in this section extends the idea of public sector size by accounting for the average human capital of public workers. Countries may have only a small share of workers allocated into the public sector, but these workers may represent a large share of highly educated workers in the economy. Fig. 5 suggests that poorer countries "compensate" for small share of public workers with high-educated workers. Then, although these countries seem to have not so sizeable public sectors, they are using a relevant share of high-productivity workers and hindering their growth rates.

6 Conclusion

As government size increases, more and more resources are reallocated from the private to the public sector, reducing the size of the market and the size of firms in terms of production. Smaller firms do not have incentives to invest in new methods of production, harming the economic growth of the countries. Therefore, the public sector must be large enough to provide public goods used to reduce firms' operational costs without crowding out many workers from the private sector. The same intuition follows when the labor market is composed of efficient units of labor. However, in this case, governments can crowd out the labor market by either increasing the number of workers or the average years of schooling of the public workers. The latter channel is relevant since poorer countries tend to allocate more educated workers in the public sector.

This paper studied an economy where governments reduce the operational costs of firms and use workers available in the economy. The main analytical result from the model is that there is an inverted U-shaped relationship between public employment and economic growth. I show that there is evidence of such a nonlinear relationship using growth regressions. Numerical exercises show that the model can capture this relationship almost as good as a quadratic fit when we match only its peak. For the exercise where the parameters are chosen to match the French experience, the model still performs well but underestimates growth rates. This discrepancy occurs due to the higher cost of technology adoption faced by France compared with the countries in the sample. Nonetheless, the central nonlinear relationship between the share of public employees and economic growth is unchanged.

A Appendix

A.1 Regression Tables

Table A.1: Regression Tables – Figs. 2 and 3a

	1	2	3
	Panel A: Average Growth (2000-2016)		
s^p	23.6846***	35.7445***	31.5827***
	(8.3080)	(8.7775)	(9.6250)
s^p-squared	-43.3530**	-62.7314***	-58.0576***
	(17.5594)	(17.7933)	(19.5003)
ln(GDP per capita$_{2000}$)		-0.9061***	-1.5753***
		(0.3010)	(0.3308)
ln(Avg Human Capital$_{2000}$)			3.5144***
			(1.0541)
Intercept	1.3931**	7.6176***	11.2459***
	(0.6818)	(2.2288)	(2.4576)
N	97	97	82
R2	0.11	0.19	0.30
	Panel B: Public Goods		
ln(s^p)	2.1228***	1.1349***	1.1400***
	(0.2148)	(0.2623)	(0.2643)
ln(GDP per capita$_{1990}$)		1.3831***	1.4146***
		(0.2297)	(0.2338)
ln(Public Capital per capita)			-0.0408
			(0.1597)
Intercept	3.4581***	-10.2350***	-10.1595***
	(0.4341)	(2.3909)	(2.4895)
N	56	56	56
R2	0.59	0.79	0.79

Robust standard errors in parentheses. s^p is the share of public workers whose information is from WWBI. GDP per capita and the index of human capital are from PWT9.1. The measure of public goods is from Desmet et al. (2017). It is the principal component of 8 indicators: infant mortality, measles immunization rate, hospital beds per capita, log school attainment, percentage access to improved sanitation, percentage access to improved water, a measure of infrastructure quality, and railway length per capita. The stock of public capital is from the IMF investment and capital stock database (ICSD). * p-value < 0.1, ** p-value < 0.05, *** p-value < 0.01.

Table A.2: Regression Tables – Fig. 6

	1	2	3
		Panel A: Average Growth (2000-2016)	
$h^p s^p$	6.7701***	9.8815***	11.0511***
	(2.0020)	(2.2168)	(2.7077)
$(h^p s^p)^2$	-3.0961***	-4.4580***	-6.2422***
	(1.0990)	(1.2639)	(1.5813)
ln(GDP per capita$_{2000}$)		-0.9091***	-1.5589***
		(0.3331)	(0.3550)
ln(Avg. Human capital$_{2000}$)			3.6411***
			(1.1438)
Intercept	1.5938**	7.9815***	10.7673***
	(0.6055)	(2.5336)	(2.6944)
N	76	76	70
R2	0.17	0.24	0.37
		Panel B: Public Goods	
$\ln(h^p s^p)$	2.0389***	1.6929***	1.2075***
	(0.2109)	(0.2097)	(0.2442)
ln(Public Capital per capita$_{2000}$)		0.5436**	-0.0316
		(0.2100)	(0.1517)
ln(GDP per capita$_{2000}$)			1.3021***
			(0.2400)
Intercept	1.0787***	-3.6422**	-10.3900***
	(0.2184)	(1.7931)	(2.3851)
N	48	48	48
R2	0.62	0.68	0.80

Robust standard errors in parentheses. $h^p s^p$ is the share of public workers (s^p) adjusted by their average human capital (h^p) whose information is from WWBI. GDP per capita and the index of human capital are from PWT9.1. The measure of public goods is from Desmet et al. (2017). It is the principal component of 8 indicators: infant mortality, measles immunization rate, hospital beds per capita, log school attainment, percentage access to improved sanitation, percentage access to improved water, a measure of infrastructure quality, and railway length per capita. The stock of public capital is from the IMF investment and capital stock database (ICSD). It is the difference between the democracy and autocracy indexes and ranges from -10 to 10 being 10 the most democratic regime and -10 the most autocratic regime. * p-value < 0.1, ** p-value < 0.05, *** p-value < 0.01.

A.2 Proofs

Proof of Proposition 1 Define $\Gamma(\gamma_v) \triangleq [2\beta\phi(1+\gamma_v)-1]e^{\beta\phi v}[1+\phi(1+\gamma_v)]^{\beta}$ and

$$\Lambda(s^p) \triangleq \left[\frac{2(1-s^p)N}{(\kappa \circ F)(s^p)}\right]^{\beta},$$

such that $G(\gamma_v, s^p) = \Gamma(\gamma_v) - \Lambda(s^P) = 0$. Using the implicit function theorem

$$\frac{\partial \gamma_v}{\partial s^p} = \frac{\Lambda_{h^p}(h^p)}{\Gamma_{\gamma_v}(\gamma_v)}.$$

One can show that

$$\Gamma_{\gamma_v}(\gamma_v) = e^{\beta\phi\gamma_v}\left[2\phi\beta^2\varepsilon_v^{\beta-1}(\varepsilon_v^2-1) + \phi\beta\varepsilon_v^{\beta-1}(\varepsilon_v-1)\right] > 0,$$

where $\varepsilon_v = (1+\gamma_v)\phi + 1$, and

$$\Lambda_{s^p}(s^p) = -\beta\Lambda(s^p)\left[\frac{1}{1-s^p} + \eta^{\kappa}\eta^{f}\frac{1}{s^p}\right],$$

where $\eta^{\kappa} < 0$ is the elasticity of operational cost with respect to public goods and $\eta^{f} > 0$ is the elasticity of public goods with respect to s^p. Assuming that $|\eta^{\kappa}| \leqslant B_0 < \infty$ for all g and $|\eta^{f}| \leqslant B_1 < \infty$ for all $s^p \in [0,1]$, then $\frac{\partial \gamma_v}{\partial s^p} > 0$ as $s^p \to 0$ and $\frac{\partial \gamma_v}{\partial s^p} < 0$ as $s^p \to 1$. Also, given that our functions are continuous with respect to s^p, $\frac{\partial \gamma_v}{\partial s^p}$ is also continuous with respect to s^p which allows as to use the intermediate value theorem to state that there is s^p such that $\frac{\partial \gamma_v}{\partial s^p} = 0$.

Proof of Proposition 2 The proof of proposition 2 is identical to the proof of proposition 1 when we redefine

$$\Lambda(h^p s^p) \triangleq \left[\frac{2(h-h^p s^p)N}{(\kappa \circ F)(h^p s^p)}\right]^{\beta}.$$

Then one can apply the implicit function theorem to $G(\gamma_v, h^p s^p) = \Gamma(\gamma_v) - \Lambda(h^p s^P) = 0$, to find the derivative of γ_v with respect to $h^p s^p$.

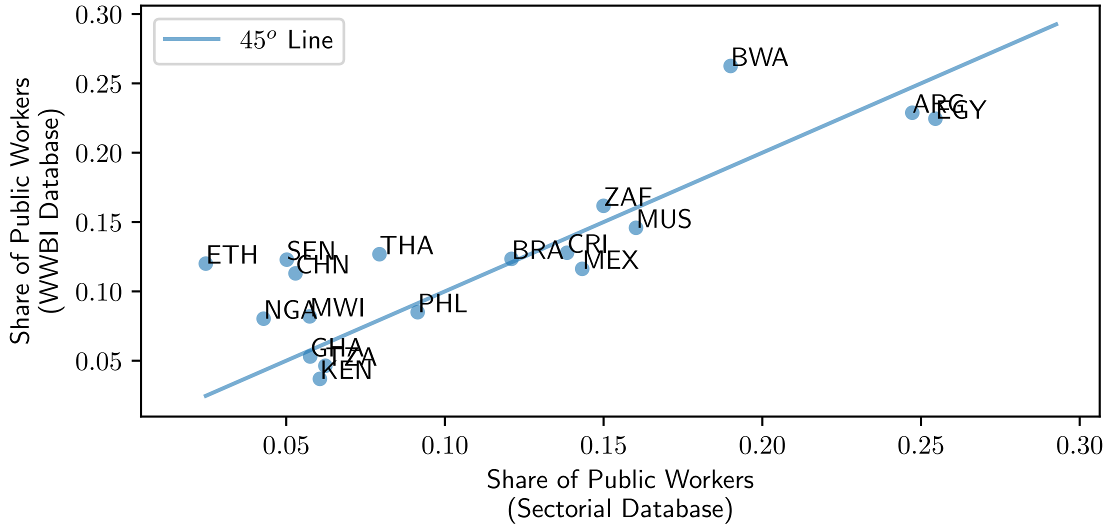

Figure A.1: share of public workers in two databases

Note: In both databases, the share of public workers is computed as the number of public employees over total employment. The values presented are the average between 2000 and 2016. The sectoral data considers public employees as a worker in one of the following sectors: public administration, education, or health. The WWBI database considers public employees as workers of state-owned institutions.

A.3 Sectoral Data

In Section 3, I provide evidence of an inverted U-shaped relationship between economic growth the share of public workers. In this section, I provide the same evidence using an alternative database for public employment. Public workers are workers allocated in the production of government services (Timmer et al., 2015). Government services are provided by workers in the following ISIC categories: *public administration and defense; compulsory social security*, *education*, and *health and social work*. Fontaine et al. (2020), for example, also defines public employment using industry allocation. The limitation is that the public sector does not necessarily provide education and health services. The relationship between both databases is presented in Fig. A.1. Although only a few countries overlap across databases, one can see that the values are similar.

As in Section 3, the share of public workers s^p is defined as the ratio to total employment. Data on real output per capita is from PWT9.1. Fig. A.2 shows the average growth rates of countries in the sample between 1970 and 2010 and its relationship with the average share of public workers in the same period.[15] The sample contains 32 countries, developed and de-

[15]The main picture is unchanged when I use the periods 1980-2010 or 1990-2010.

veloping, from Africa, Asia, Europe, and the Americas. On average, countries allocate 13.5% of total employment to the provision of government services. Sweden has the largest share with 32% of public workers, whereas Ethiopia allocates the least with only 2.3% of the labor force providing public goods. In terms of average growth, the average of the sample is 2.7% per year. Botswana stands out as the fastest-growing country with 7.5% per year, whereas Malawi performed a negative average growth rate of -0.3%.

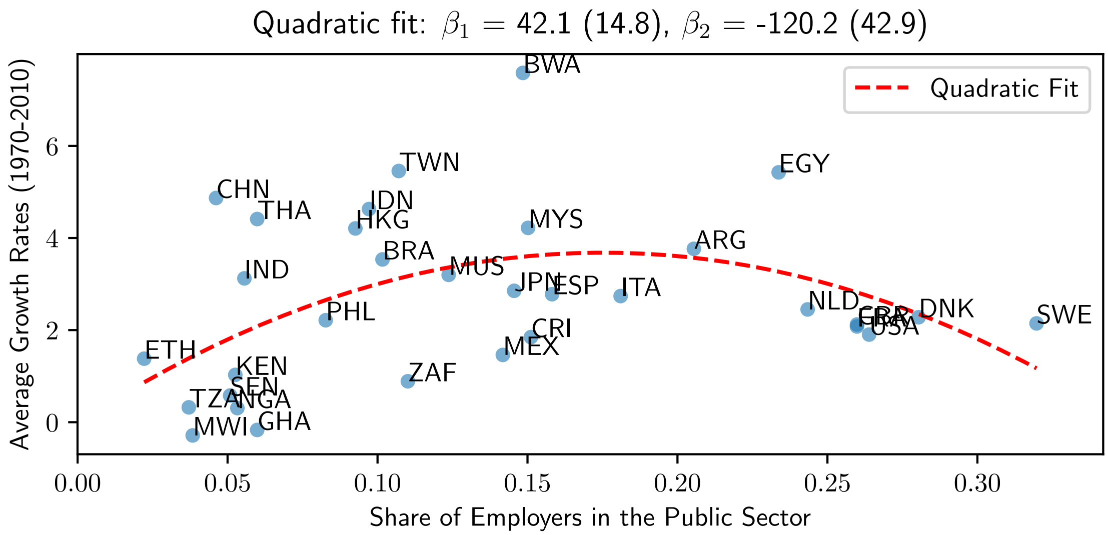

Figure A.2: Economic growth, public goods, and adjusted share of public workers

Note: Data on public employment is from the sectoral database (Timmer et al., 2015). Growth of GDP per capita is from PWT9.1 (Feenstra et al., 2015). The values are averages of 1970 to 2010. The quadratic fit fits $\gamma_i = \alpha + \beta_1(s_i^p) + \beta_2(s_i^p)^2$. Robust standard errors in parentheses.

Fig. A.2 highlights that countries with low shares of workers allocated to the public sector experienced a slow average growth of output per capita between 1970 and 2010. As the share of public employment increases, the growth rates also increase but at diminishing rates. At some level, increasing the share of public employment seems to be detrimental to economic growth. It characterizes the inverted U-shaped relationship stressed the quadratic fit in the figure. This nonlinear relationship stands when I control for initial conditions often considered to determine growth. Specifically, initial log of output per capita and initial average human capital. Results are provided in Table A.1. The simple correlation depicted in Fig. A.2 predicts an optimal size of public employment equal to 17.5%. When I control for the initial level of development, this number increases to 21.5%

Table A.1: Regression Tables – Fig. A.2

	1	2	3
	Panel A: Average Growth (1970-2010)		
s^p	42.1265***	58.9223***	55.6450***
	(14.8029)	(11.8403)	(12.8104)
s^p-squared	-120.2216***	-136.8030***	-137.3392***
	(42.9307)	(33.5960)	(34.2679)
ln(GDP per capita_{1970})		-1.3145***	-1.8541***
		(0.2893)	(0.4613)
ln(Avg. Human capital_{1970})			2.7221
			(1.6164)
Intercept	-0.0090	9.0885***	12.6417***
	(0.9948)	(2.2311)	(3.6367)
N	32	32	32
R2	0.21	0.51	0.54
	Panel B: Public Goods		
$\ln(s^p)$	2.9620***	1.3749***	1.1108***
	(0.2993)	(0.3239)	(0.3734)
ln(Public Capital per capita)		1.0880***	0.7569**
		(0.1864)	(0.3011)
ln(GDP per capita_{1990})			0.5510*
			(0.3099)
Intercept	-7.5960***	-13.1059***	-14.3687***
	(0.8203)	(1.0218)	(1.0914)
N	27	27	27
R2	0.77	0.91	0.92

Robust standard errors in parentheses. s^p is the share of public workers whose information is from the sectoral database (Timmer et al., 2015). GDP per capita and the index of human capital are from PWT9.1. The measure of public goods is from Desmet et al. (2017). It is the principal component of 8 indicators: infant mortality, measles immunization rate, hospital beds per capita, log school attainment, percentage access to improved sanitation, percentage access to improved water, a measure of infrastructure quality, and railway length per capita. The stock of public capital is from the IMF investment and capital stock database (ICSD). * p-value < 0.1, ** p-value < 0.05, *** p-value < 0.01.

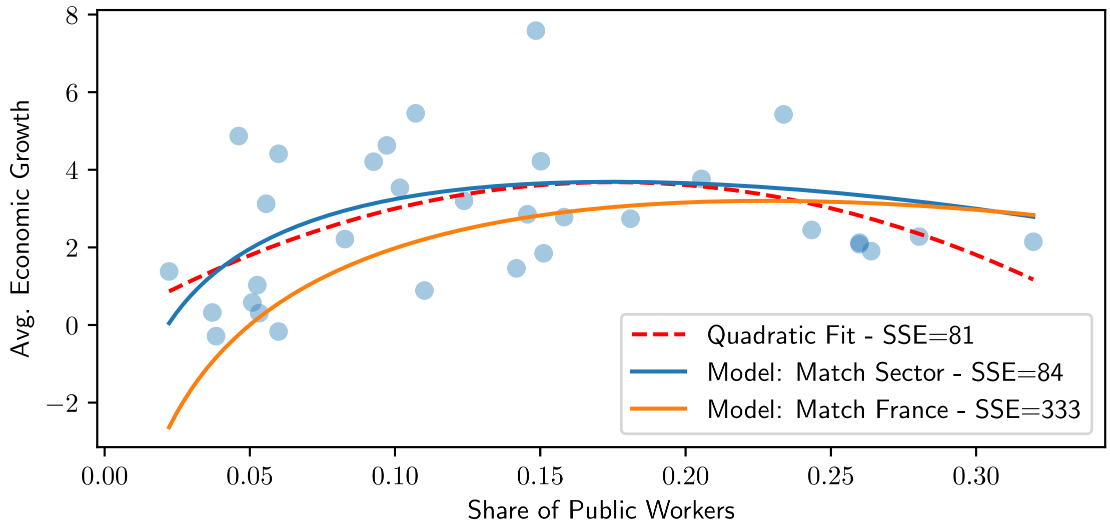

Figure A.3: Numerical Exercise

Note: Same data as in Fig. 2. The *match WWBI* model has α=1, β=0.5, z=1, N=1, ϕ=1.50, η=-0.38. *Match France* has α=1, β=0.5, z=1, N=1, ϕ=1.51, η=-0.41.

Fig. A.3 shows how the model adjusts to this sample. As before, it has an exercise in which the choice of η and ϕ match the optimal size of government estimated by the quadratic fit. I call this exercise *macth sector*. I also repeat the exercise that seeks to match the French experience. However, the optimal size of public employemnt is take from the sectoral data rather them from Rose and Page (1985). In this case, η is chosen to match a share of public workers equal to 22.8%, whereas ϕ matches the optimal growth 3.2% estimated by Facchini and Melki (2013). The figure shows that the model captures the main nonlinear pattern in the data. The *match sector* exercise has the SSE very close to the quadratic fit, although it overestimates the growth of richer countries. It is the outcome of the fixed ϕ. The *match France* exercise performs worse, relatively to the quadratic given that the optimal is different. However, the main nonlinear relationship is unchanged.

A.4 The model with efficient units of labor

Government In this version of the model, the production of public goods is given by $F(h^p s^p)$, where h^p is the average human capital of the workers in the public sector. Now w^g is the wage to efficient units of labor such that the government's balanced budget is

$$w^g h^p s^p N = \tau Y, \tag{A.1}$$

where Y is the aggregate income and N is the labor force.

Firms Firms also have similar production functions. However, they utilize H_v units of efficient labor, which is the only input of production. As in the benchmark model, efficient units of labor can be used either in production activities or in nonproduction activities such that

$$Q_v = A_v [H_v - \kappa(g) e^{\phi \gamma_v}], \tag{A.2}$$

where $\kappa(g)$ and $e^{\phi \gamma_v}$ have the same interpretation as in the benchmark model, but in terms of efficient labor. The variety-v firm's profit is given by

$$\Pi_v = p_v C_v - w^x H_v, \tag{A.3}$$

which leads to the same first-order conditions, where w^x is the payment to efficient units of labor. The first-order condition with respect to prices yields

$$p_v = \frac{w^x}{A_x(1+\gamma_v)} \frac{\varepsilon_v}{\varepsilon_v - 1}, \tag{A.4}$$

whereas the first order condition for the size of technology innovations is

$$-\phi \kappa(g) e^{\phi \gamma_v} + \frac{C_v}{A_x(1+\gamma_v)^2} \leqslant 0. \tag{A.5}$$

Households and Aggregate Demand In this version of the model, households are heterogeneous in their labor productivity and their most preferred variety. As in the benchmark model, there is a continuum of measure N of varieties uniformly distributed in a circle with circumference one. In each variety, there is a continuum of measure one of households. They draw their labor productivity, h, from $\mathcal{H}$ according to the probability density function $q(\cdot)$ such that total stock of human capital in the economy is

$$H = \int_0^N \int_{h\in\mathcal{H}} hq(h)dhd\nu.$$

The draws of the labor productivity variety independent, which means that high-productivity workers are as likely to prefer one kind of good as low-productivity workers. This assumption abstracts from any unbalanced increase in the number of high-productivity-workers-biased goods.

Now, the labor productivity of the household affects the consumption decision such that the utility function in contingent on variety and labor productivity h:

$$U_{\tilde{\nu},h}\left(\{c_\nu(h)\}_{\nu\in V}, a(h)\right) = \max_{\nu\in V}\left[\frac{c_\nu(h)}{1+d^{\beta}_{\nu\tilde{\nu}}}\right], \tag{A.6}$$

subject to

$$\sum_{\nu\in V} p_\nu c_\nu(h) \leqslant (1-\tau)\left[a(h)w^g + (1-a(h))w^x\right]h.$$

The result that individuals choose only their most-preferred variety still holds such that total expenditure is

$$p_{\nu'}c_{\nu'} = \begin{cases} (1-\tau)w^g h & \text{if } w^g > w^x \\ (1-\tau)w^x h & \text{if } w^g \leqslant w^x \end{cases} \tag{A.7}$$

The equilibrium in this economy is a symmetric Nash equilibrium where all m varieties are equally spaced around the varieties circle so that $d = 1/m$. Then, the household who is indifferent between variety k and an adjacent variety j is the one located at a distance d^i_k from k, such that

$$p_j[1+(d-d^i_k)^\beta] = p_k[1+(d^i_k)^\beta], \tag{A.8}$$

which leads to the aggregate demand of variety k is

$$C_k = \frac{(1-\tau)2d_k^i}{p_k}\left[\int_0^N \int_{h\in\mathcal{H}} a(h)w^g hq(h)dhdv + \int_0^N \int_{h\in\mathcal{H}} (1-a(h))w^x hq(h)dhdv\right]. \tag{A.9}$$

Again, the aggregate demand for variety k depends on the measure d_k^i, which, in turn, changes with the price of the variety k and the price-elasticity of demand is still

$$\varepsilon_k = 1 + \frac{(1+d_k^i)^\beta[1+(d-d_k^i)^\beta]}{\beta(d-d_k^i)^{\beta-1}[1+(d_k^i)^\beta]d_k^i + \beta(d_k^i)^\beta[1+(d-d_k^i)^\beta]}. \tag{A.10}$$

Definition 2 (Symmetric Equilibrium with efficient labor units)**.** *The symmetric equilibrium is a vector of variety-dependent variables* $\{p_v, d_v^i, \varepsilon_v, Q_v, \gamma_v, C_v\}_{v\in V}$ *for all* $v \in V$ *and a vector* $\{d, w^x, w^g, \tau\}$ *that, given* $\{H, N, s^p, h^p\}$*, satisfy:*

1. *Firm maximization conditions Eqs.* (A.4) *and* (A.5)*, and the zero profit condition [Eq.* (A.3)*=0] for all* $v \in V$*;*
2. *The aggregate demand Eq.* (A.9) *for all* $v \in V$*;*
3. *The government balanced budget condition Eq.* (A.1)*;*
4. *The symmetric equilibrium price condition Eq.* (A.8) *for all* $v \in V$*;*
5. *The elasticity Eq.* (A.10) *for all* $v \in V$*;*
6. *The good's market clearing:* $Q_v = C_v$ *for all* $v \in V$*;*
7. *Labor market equilibrium wages:* $w^x = w^g$*; and*
8. *The labor market clearing:* $\frac{1}{d}H_v = (H - h^p s^p N)$ *for all* $v \in V$.

Eqs. (A.1) and (A.9) and $w^x = w^g$ yield the aggregate demand as a function of efficient units of labor.

$$C_k = \frac{2d_k^i w^x}{p_k}(H - h^p s^p N). \tag{A.11}$$

Note that if all workers have labor productivity equal to unit, we are back to the benchmark model. As in the benchmark model, the symmetry conditions $p_j = p_k$ for all $j, k \in V$ and $d = 2d_k^i$ together with Eq. (A.10) yield the equilibrium equation for the price-elasticity of demand

$$\varepsilon_k = 1 + \frac{1}{2\beta}\left(\frac{2}{d}\right)^{\beta} + \frac{1}{2\beta}, \tag{A.12}$$

and the number of firms can be found using the final goods market clearing condition together with the firm's first-order condition Eq. (A.5) and the private production function such that $H_k = \kappa(g)e^{\gamma_k \phi}\varepsilon_k$. It can be used together with the labor market clearing condition to find

$$m = \frac{1}{d} = \frac{(H - h^p s^p N)}{H_k} = \frac{(H - h^p s^p N)}{\kappa(g)e^{\gamma_k \phi}\varepsilon_k}. \tag{A.13}$$

Government size affects the number of firms in two ways: the first is the crowding-out on the labor market mentioned above; the second is by reducing the costs of production in terms of skilled labor. With reduced costs, there will be more firms in the economy, what increases the price-elasticity of demand in Eq. (A.12).

The zero profit condition together with equations Eqs. (A.5), (A.12) and (A.13) give us

$$[2\beta\phi(1+\gamma_k) - 1]e^{\beta\phi\gamma_k}[1 + \phi(1+\gamma_k)]^{\beta} = \left[\frac{2(H - h^p s^p N)}{(\kappa \circ F)(h^p s^p)}\right]^{\beta}. \tag{A.14}$$

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